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## One-to-one coordination algorithm for decentralized area partition in surveillance missions with a team of aerial robots

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**Abstract** This paper presents a decentralized algorithm for area partition in surveillance missions that ensures information propagation among all the robots in the team. The robots have short communication ranges compared to the size of the area to be covered, so a distributed one-to-one coordination schema has been adopted. The goal of the team is to minimize the elapsed time between two consecutive observations of any point in the area. A grid-shape area partition strategy has been designed to guarantee that the information gathered by any robot is shared among all the members of the team. The whole proposed decentralized strategy has been simulated in an urban scenario to confirm that fulfils all the goals and requirements and has been also compared to other strategies.

### 1 INTRODUCTION AND RELATED WORK

The European Project EC-SAFEMOBIL<sup>1</sup> is devoted to the development of sufficiently accurate common motion estimation and control methods and technologies in order to reach levels of reliability and safety to facilitate unmanned vehicle deployment in a broad range of applications. One of the applications of the project includes the distributed safe reliable cooperation and coordination of many high mobility entities. The aim is to precisely control hundreds of entities efficiently and reliably and to certify developed techniques to support

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<sup>1</sup> <http://www.ec-safemobil-project.eu/>

the exploitation of unmanned platforms in non-restricted areas. Two scenarios have been chosen for the validation of the developments: industrial warehousing involving a large number of autonomous vehicles and surveillance also involving many mobile entities. This paper is focused on the latter scenario.

Surveillance missions with unmanned aerial vehicles (UAVs) have been widely studied in different contexts [19]: automated inspection, search and rescue missions, military applications, etc. A decentralized solution using a large-scale team of aerial robots in the EC-SAFEMOBIL surveillance scenario is proposed in this paper. The application of multi-UAV systems allows to accomplish them with robustness against failures, higher spatial coverage and an efficient deployment [18, 23, 16].

Area surveillance missions can be addressed using a frequency-based approach, where the objective implies to optimize the elapsed time between two consecutive visits to any position which is known as the refresh time. This approach has been used by many authors, obtaining solutions to guarantee an uniform frequency of visits as in [11], or the maximal minimum frequency as in [6]. The obtained solution is a deterministic motion plan for each vehicle. Some authors, as in [5], address the patrolling problem in adversarial settings applying a probabilistic approach because with a deterministic solution, intelligent intruders could learn the strategy. A frequency-based approach is also followed in [9], which defines and compares different partitioning and cyclic patrolling strategies. Authors of [21] analyze the *refresh time* and *latency* in area coverage problems with multiple robots using different approaches. A partitioning method is proposed in [22] to monitor a set of positions with different priorities.

Reference [13] proposes an on-line algorithm where the area to cover is initially unknown that solve the problem for multi-robot systems using Voronoi spatial partitioning. An off-line algorithm, where the area to cover is known *a priori*, is proposed in [15]. Authors creates an spanning tree to generate a coverage path around it. The most well known off-line coverage path planning is called *Boustrophedon Cellular Decomposition* and was presented in [10]. It proposes to divide the whole area into smaller sub-areas which can be covered with a simple *back and forth* method. In our work, a *back and forth* method with some additional modifications to obtain a closed coverage path is proposed. These modifications are directed to keep periodical data interchange between neighbors even under limited communication ranges.

This paper proposes an area partitioning strategy to solve the problem for irregular areas and heterogeneous UAVs. The whole area is divided into non-overlapping sub-areas, each one monitored by an aerial robot using an efficient path, i.e. all the positions in the area are observed while the path is traveled, minimizing the total path length. Each robot covers an area with a size related to its motion and sensing capabilities, minimizing the time to cover the whole area. A similar strategy was presented in [4] to solve the area patrolling problem with a team of homogeneous UAVs and rectangular areas. On other hand, in [1] the problem with irregular areas and heterogeneous UAVs is solved using a path partitioning strategy. A single coverage path

is created to monitor the whole area and the path is divided in segments that are allocated to the different UAVs. Other authors as [20] propose cyclic strategies where all the robots patrol the same closed coverage path in the same direction and equally spaced through it. This strategy offers theoretically optimal results from a frequency-based approach with homogeneous robots. However, in scenarios with constrained communications, the robots could not share the required information.

On the other hand, the *one-to-one coordination* technique allows the system to obtain the whole coordination from local decisions and information even when the communication range of the robots is short compared to the size of the area where the mission should be executed. The resulting system is scalable, because each UAV only needs information from nearby neighbors. A decentralized approach offers robustness and dynamism, in a way that each UAV can quickly self-adapt its sub-area. Therefore, the system is able to perform the surveillance mission in the more efficient manner, even if the communication link with the control station is broken.

Regarding decentralized coordination, cooperation between a team of UAVs and robots to accomplish perimeter surveillance missions is approached in [17] and [3], respectively, by using the technique of *coordination variables*. *Coordination variables* are the minimum global information required by each robot to solve the problem in a coherent manner. The selection of that variables can be difficult for complex problems. In [4], *one-to-one coordination* was applied to solve a rectangular area coverage problem with a team of homogeneous UAVs. This technique implies that each pair of UAVs solves a coordination problem including only their own information. In [8], the authors use a similar technique to coordinate a team of video-cameras in surveillance missions.

Finally, it should be mentioned that the work presented here extends previous work of the authors [2] providing a deeper mathematical analysis of the main algorithm and additional simulation results.

## 2 PROBLEM STATEMENT

Let us consider an irregular area  $S \in \mathbb{R}^2$  with a surface  $A$  which has to be patrolled by a team of heterogeneous aerial robots  $Q := \{Q_1, Q_2, \dots, Q_N\}$  to detect pollution sources (see Fig. 1). There is no “a priori” information about the area, so the pollution sources can appear in any position with the same probability. Then, all the positions into the area  $S$  should be monitored at the same minimum rate.

At any time  $t$ , each aerial robot  $Q_i$  moves along the area  $S$  following a path with a motion speed  $v_i(t)$  and monitoring an area  $C_i(t)$ .

$$C_i(t) := \{r \in \mathbb{R}^2 : |r - r_i(t)| < c_i(t)\}, \quad (1)$$

where  $r_i(t) \in \mathbb{R}^2$  is the robot center projection on the plane  $z = 0$  and  $c_i(t) = z_i(t) \cdot \tan(\theta_i)$  is the actual aerial robot coverage range, with  $z_i(t)$  as its altitude and  $\theta_i$  as its angle of view.

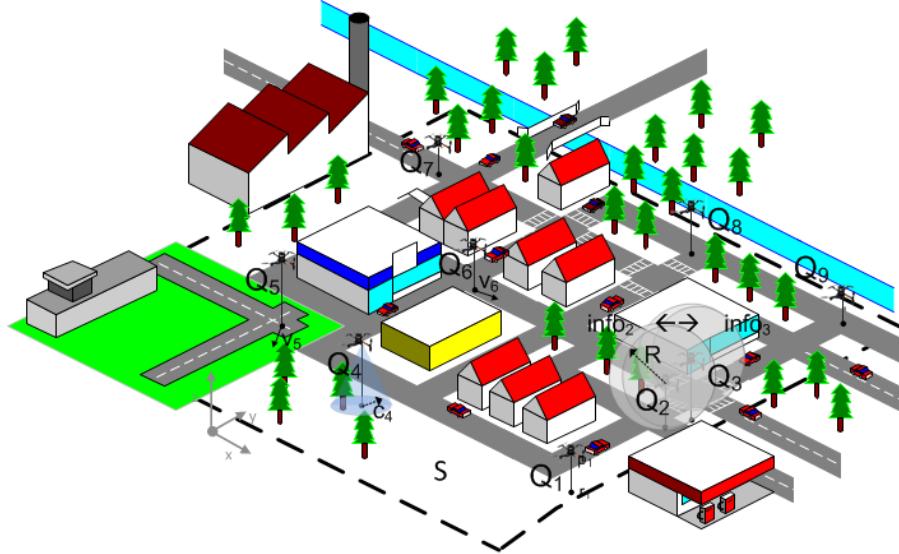


Fig. 1: A team of nine aerial robots performing a surveillance mission in an irregular urban area  $S$ . All the positions into the area  $S$  should be monitored at the same minimum rate because the threats can appear in any position with the same probability.

Each aerial robot  $Q_i$  could have different capabilities: a maximum motion speed  $v_i^{\max}$  and a maximum coverage range  $c_i^{\max}$  related to its optimal flight altitude  $h^{opt}$ . The *coverage speed*  $a_i$  can be defined as the area covered per second and can be approximated according to the coverage range  $c_i$  and the motion speed  $v_i(t)$  as

$$a_i(t) \approx 2c_i(t)v_i(t). \quad (2)$$

A communication range  $R$  for the aerial robots is also considered: two vehicles can exchange information only if they are close enough, i.e. the distance between them is less than the communication range  $R$ .

The objective is to design a cooperative patrolling strategy for minimizing both the maximal refresh time ( $T_r$ ) and the maximal time to share a detected information with the rest of the team (latency  $T_s$ ). The second objective is challenging due to the communication constraints mentioned above.

### 3 AREA PARTITIONING

To address the first objective outlined in the above section, we propose an area partitioning strategy based on the one-to-one communication technique.

A disjoint partition of the area  $S$  is considered.  $S$  divided in  $N$  non-overlapped sub-areas  $S_i$  so that

$$\begin{aligned} S_1 \cup S_2 \cup \dots \cup S_N &= S \\ S_1 \cap S_2 \cap \dots \cap S_N &= \emptyset. \end{aligned} \quad (3)$$

Each aerial robot  $Q_i$  can patrol a sub-area  $S_i$  following a different coverage closed path  $P_i$ . The minimum maximal refresh time is obtained if the robots move at their optimal altitude with their maximum speeds, and each one covers a sub-area  $S_i$  with a size of  $A_i$  related to its own maximum coverage speed:

$$A_i = a_i^{\max} \frac{A}{\sum_{j=1}^N a_j^{\max}}, \forall i = 1, \dots, N \quad (4)$$

It is easy to see that for an optimal partition, all the aerial robots spend the same time  $T$  to complete its own coverage path  $P_i$ . Let suppose, on the contrary, that there exist two different elapsed times in the optimal solution. Consider now the area with maximum elapsed time and take a neighboring area with a shorter path. Thus, by continuity, the paths can be slightly modified to improve the maximum elapsed time and this contradicts the optimality. Then, the minimum maximal refresh time will be lower limited to  $T$ .

$$T = A_i / a_i^{\max} = \frac{A}{\sum_{j=1}^N a_j^{\max}} \quad (5)$$

The area partitioning strategy should offer better result with non homogeneous aerial robots because it exploits their different capabilities: maximum speed and maximum coverage range. Other kinds of patrolling strategies does not take advantage of the better performance that can have some vehicles in the team.

In the one-to-one communication strategy, to ensure that any information detected by an aerial robot can be shared with the rest of the team implies that adjacent paths of two robots should be linked by a pair of positions near enough (closer than the communication range). Moreover, the robots should be synchronized in time when visiting these positions. This is a challenging issue addressed in Section 4. On the other hand, the maximum time to share information  $T_s$  depends on the division shape that will be also discussed in the next section.

#### 4 SYNCHRONIZATION

Typically, the usual method to achieve synchronization reduces to change the speeds of the aerial robots by a small amount relative to the nominal flight speed. Unfortunately, this simple approach is only feasible for two vehicles. For a team of cooperative robots, it requires a more delicate study.

Let us assume that the area partition is given by  $N$  non overlapped sub-areas with  $N$  non overlapped closed paths, each one traveled by a different aerial robot. A communication data link between two aerial robots is possible only if the distance between two points of their paths are closer than the

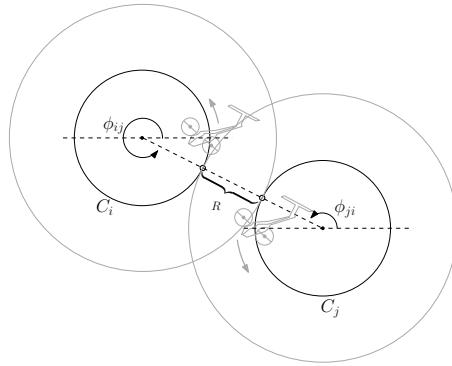


Fig. 2: The circular model. Robots  $i$  and  $j$  are in the starting position and move in the same direction.  $\phi_{ij}$  (resp.  $\phi_{ji}$ ) is the angle at which  $i$  (resp.  $j$ ) is closest to  $j$ 's trajectory (resp.  $i$ 's trajectory).

communication range  $R$  and the robots are synchronized in time when visiting these points.

Let us define a *link* between each pair of paths by two points, one for each path, with a distance between them lower than the communication range  $R$ . Then, two aerial robots are defined as *neighbors* if they have a common link. They can exchange information if they are synchronized, i.e. they pass through the link simultaneously. In order to ensure information exchange in the system, every pair of neighbors has to be synchronized.

In general, a synchronization between two neighbors cannot be guaranteed. For example, if the speeds are the same for both vehicles and the lengths of the paths are not proportionally rational, a synchronized flight is not possible. This can be fixed by adjusting the speeds but, as we mentioned before, this does not work for a team of aerial robots.

In this section a simplified model is proposed where the synchronization between a team of aerial robots can be achieved. After that, it is shown that the characterization for a solution in the simple model is the key to guarantee the information exchange in more general scenarios.

#### 4.1 The solution for unit circular paths

Suppose that all the coverage closed paths are of the same length (it can be assumed without loss of generality, otherwise the speeds can be changed to match the times). Thus, the model can be simplified by considering all the aerial robots move on unit circles. Moreover, in our simple model we assume all the robots move on unit circles in the counterclockwise direction at constant speed. With this assumption, it is given  $N$  pairwise disjoint unit circles  $C_1, C_2, \dots, C_N$  and  $N$  aerial robots  $Q_1, Q_2, \dots, Q_N$  moving on the circles in the same direction. A model with the above constraints is named here as the

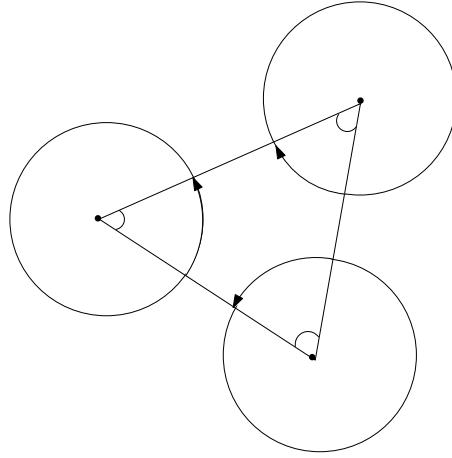


Fig. 3: An odd cycle.

*circular model.* Let  $R$  be the communication range. Two aerial robots are called *neighbors* if the smallest distance between the corresponding circles is less or equal to  $R$ . Thus, two neighbors can see each other at the smallest distance between the circles. Let us denote the position of a robot by the angle on its circle (measured from the positive horizontal axis). Let  $\alpha_i$  be the starting position of the  $i$ th robot. Furthermore, for any pair of robots,  $i$  and  $j$ ,  $\phi_{ij}$  denotes the angle at which  $i$  is closest to  $j$ 's trajectory, see Fig. 2.

Given a set of paths (unit circles), it is defined the visibility graph associated to the range  $R$  and the set of circular paths as a planar graph  $G(R) = (V, E(R))$  whose vertexes are the centers of the circles and the edges connect two centers if their distance is less or equal than  $2 + R$ .

A graph  $G(V, E)$  is *bipartite* if there are sets  $V_1, V_2 \subseteq V$  such that  $V_1 \cup V_2 = V$ ,  $V_1 \cap V_2 = \emptyset$ , and  $(u, v) \in E$  only if  $u \in V_1$ ,  $v \in V_2$  or  $v \in V_1$ ,  $u \in V_2$ . Additionally, a graph is bipartite if and only if it has no subgraph that is a cycle of odd length.

It is easy to see that an odd cycle cannot be synchronized. Let's consider an odd cycle as in Fig. 3. Since the sum of the internal angles of the triangle is less than  $2\pi$ , a scheduling is not possible. However, for even cycles, the synchronization of the model is possible.

In [7], it has been proved the following result.

**Theorem 1** *A team of mobile robots in the circular model can be synchronized if and only if the visibility graph is a bipartite graph. Moreover, the condition  $\phi_{ij} = \pi + \phi_{ji}$  for every pair of neighbors  $i \neq j$ , ensures synchronization of the team.*

As a consequence of Theorem 1, if the visibility graph is a grid-shape configuration, it can be synchronized to share information and then, it is a simple suitable area partition. Given a  $m \times n$  grid-shape area division, with

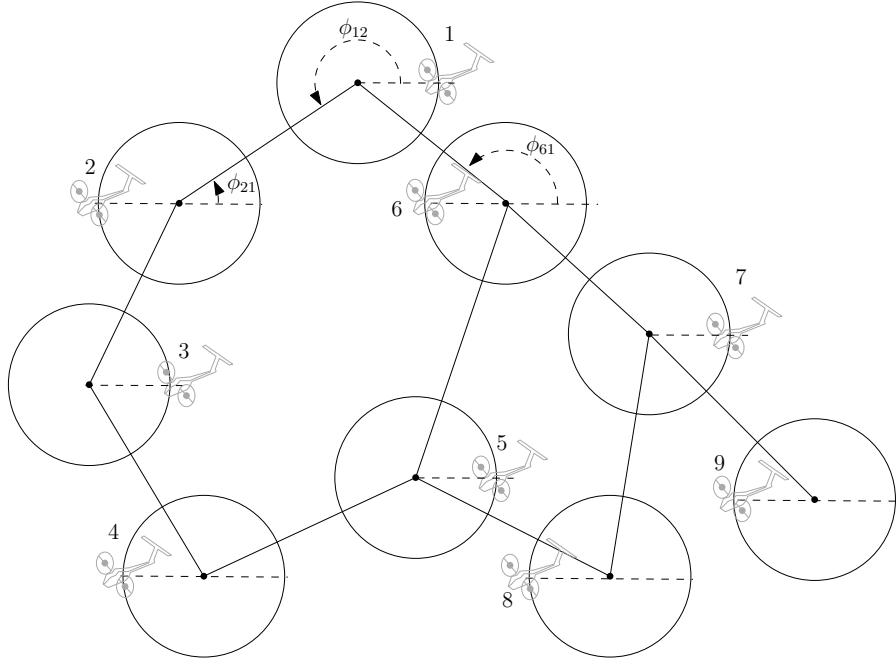


Fig. 4: A synchronized scheduling in the circular model. All cycles are even. The aerial robots are in the starting positions.

$m$  rows and  $n$  columns, it was shown in [2] that an upper-bound for the time to share any detected information with the rest of the team is given by

$$T_s \leq 5T/4 + (n + m - 4)T/2, \quad (6)$$

where  $T$  is the maximum time since a robot detects any event until it is communicated with its neighbors.

Finally, Fig. 4 shows a non-grid configuration with a synchronized scheduling.

#### 4.2 Generalization

As a consequence of the above theoretical results, it is possible to guarantee that each pair of neighbors pass through the common link simultaneously under the following *synchronization conditions*: The trajectories are equal-size circles; all aerial robots travel in the same direction and spend the same time to travel the tour; the visibility graph is bipartite.

Now, it is explored how to relax the above constraints to address a more general model. A suitable strategy would be to adapt both the trajectories and connections of the aerial robots so that above synchronization conditions are satisfied. Some examples are considered here.

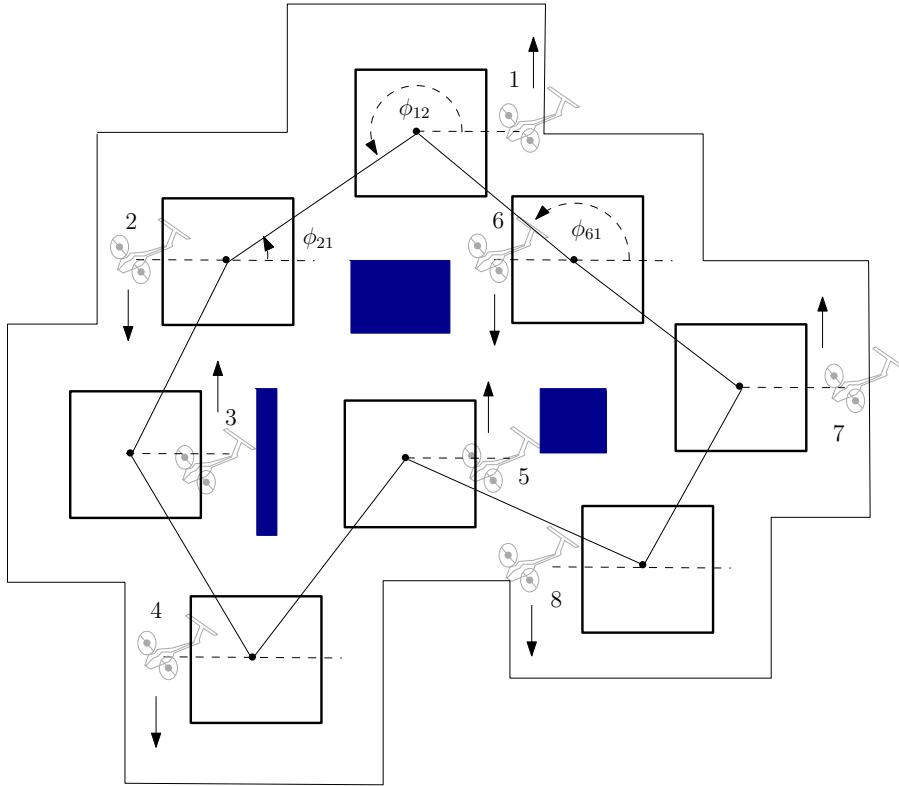


Fig. 5: A synchronized system for non-circular trajectories in an orthogonal region with obstacles.

*Non-circular paths:* Let us assume a bipartite visibility graph associated to a system of  $N$  non-circular periodic trajectories where the aerial robots travel with the same speed in the same direction. Some constraints on the paths can be considered to ensure synchronization. For instance, if the paths are boundaries of geometric shapes that are symmetrical with respect to a point (center), the synchronization can be guaranteed. In this case, the condition of Theorem 1 is satisfied and the starting positions of the robots can be located by the rule  $(\alpha, \alpha + \pi)$  for every pair of neighbors. An example is illustrated in Fig. 5. Notice that since the links connect the centers, they are not necessarily located at the closed pair between the corresponding paths.

*Non-bipartite visibility graph:* For a given area partition whose visibility graph is non bipartite a synchronized surveillance can not be scheduled. Thus, one possible approach is to find a bipartite subgraph with the maximum number of possible communication links, that is, to compute the *maximum bipartite subgraph*. Finding a bipartite subgraph with the maximum number of edges is a classical NP-complete problem [12]. However a maximum bipartite subgraph of a planar graph can be found in polynomial time [14]. Since the visibility

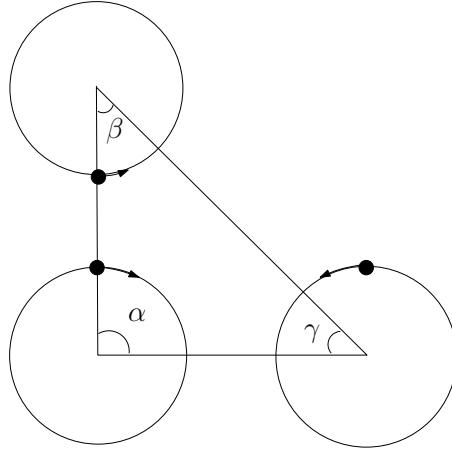


Fig. 6: An odd cycle is synchronized by changing the traveling direction. The condition  $\alpha = \beta + \gamma$  allows to solve the problem.

graph in the surveillance scenario is planar (the links do not cross each other), it is possible to adapt some algorithms from the literature to our problem. Many of them are based on the reduction of the MBS-problem to the *maximum cut problem*. See, for example [14], where the maximum cut problem is solved by means of the *maximum weighted matching problem*.

However, in practical situations where the number of aerial robots is not huge, an approximation of the maximum bipartite subgraph can be easily obtained by removing odd cycles in the planar graph. In fact, the robots can decide online which links are removed after information exchange.

*Relaxing the unidirectional traveling:* Under some conditions, an odd cycle can be synchronized if the both traveling directions are allowed. For example, in Fig. 6, the sum of the internal angles is less than  $2\pi$  but  $\alpha + \beta + (2\pi - \gamma) = 2\pi$  and the problem can be solved by changing the direction of one robot.

#### 4.3 Pseudo-symmetric coverage path

The second condition mentioned previously to keep a complete synchronization between the aerial robots is that all the paths were symmetric with respect to their own center. This condition can be not possible if the robots cover irregular areas with different shapes.

However, it is possible to ensure synchronization even with no symmetric paths assuming some extra conditions. Given a grid-shape graph, each path should have four possible link positions. Consider a non symmetric closed coverage path for each sub-area (node), so that the distance between consecutive link positions is the same, and define it as *pseudo-symmetric path*. Hence, if all the aerial robots take the same time to cover their paths, it is possible to

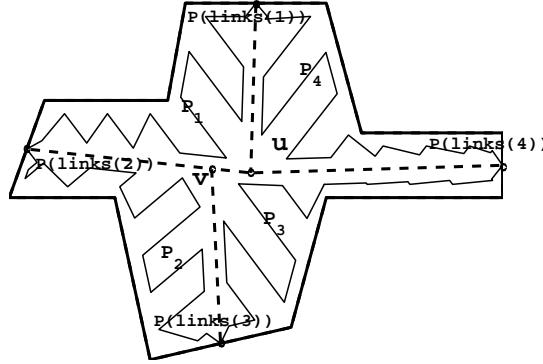


Fig. 7: Coverage closed path computed by the path generator. Thick lines define the area to cover. Narrow lines correspond to the *back and forth* paths.

ensure synchronization if starting position of neighbor robots are non consecutive link positions. Then, if  $Q_i$  starts its motion in its own first link position, all their neighbor robots start in their own third link position.

The authors define in [1] a quality index to compare the length of a coverage path with respect to the theoretically optimal according to the coverage range. Let us assume that all the generated paths have a perfect quality index equal to one. In this case, any pair of areas with the same size could be covered by paths of the same length.

It is proposed a path generator that divides the sub-area to cover in four polygons with the same area, such as each one has a pair of consecutive link positions as two of its vertices. Then, a simple *back and forth* strategy is used to generate the coverage path for each sub-area. Given a sub-area  $S_i$ , it is defined as a vector  $P$  of counterclockwise ordered points which defines the area boundary. At the algorithm implementation level, a vector  $\text{links}$  stores the indexes of link positions. Therefore,  $P(\text{links}(k))$  is the  $k^{\text{th}}$  link position, with  $1 \leq k \leq 4$ . It is assumed that exists a function  $A(P)$  which computes the size of  $S_i$  defined by vector  $P$ . Algorithm 1 shows the proposed coverage path generation method.

Assuming that the generated paths have a perfect quality index, the four paths lengths are equal. Therefore, joining the four paths, an aerial robot which moves with a constant speed would take the same time to move between any pair of consecutive link positions. Figure 7 shows how the presented path generator creates a pseudo-symmetric path to cover an irregular area.

## 5 DECENTRALIZED IMPLEMENTATION

Given an initial simple grid division of an area  $S$  using equally spaced horizontal and vertical lines, each aerial robot can initialize its own variables, see

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**Algorithm 1** Area coverage path generation for closed synchronized paths. A simple back and forth method is used to generate a different coverage path for the areas defined by  $P_1$ ,  $P_2$ ,  $P_3$  and  $P_4$ . The four generated paths are joined to create an only closed coverage path for the area defined by  $P$ .

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Each aerial robot receives the vectors  $P$  and links  
 Each aerial robot computes the area size to cover  $A_i = A(P)$   
 Two new polygons  
 $P_1 = [P(\text{links}(1) : \text{links}(2); u)]$   
 $P_2 = [P(\text{links}(2) : \text{links}(3); u)]$   
 are defined using a unknown point  $u$  interior to  $P$  such that  
 $A(P_1) = A_i/4$   
 $A(P_2) = A_i/4$   
 $u \in P$   
 A new polygon  $V = [P(\text{links}(3) : \text{links}(4); u)]$  is defined  
**if**  $A(V) = A_i/4$  **then**  
 $P_3 = V$   
 $P_4 = [P(\text{links}(4) : \text{links}(1); u)]$   
**else if**  $A(V) > A_i/4$  **then**  
 Two new polygons  
 $P_3 = [P(\text{links}(3) : \text{links}(4)); v]$   
 $P_4 = [P(\text{links}(4) : \text{links}(1)); u; v]$ ,  
 are defined such that  
 $v \in [P(\text{links}(3)); u]$   
 $A(P_3) = A(P_4) = A_i/4$   
**else**  
 Two new polygons  
 $P_3 = [P(\text{links}(3) : \text{links}(4)); v; u]$   
 $P_4 = [P(\text{links}(4) : \text{links}(1); v],$   
 are defined such that  
 $v \in [P(\text{links}(4)); u]$   
 $A(P_3) = A(P_4) = A_i/4$   
**end if**

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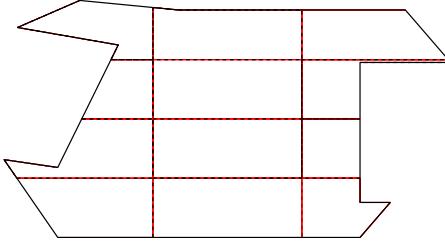


Fig. 8: Initial  $4 \times 3$  grid-shape area division.

Fig. 8. Each robot has an initial area  $S_i$  to cover and initial link positions common with its neighbors, and can generate its own coverage path.

However, for irregular areas or non homogeneous team of aerial robots, that initial division is not efficient. Some robots take longer times than others to cover their areas using their maximum capabilities. Then, some of them would have to slow down their motions to keep synchronization and the maximum

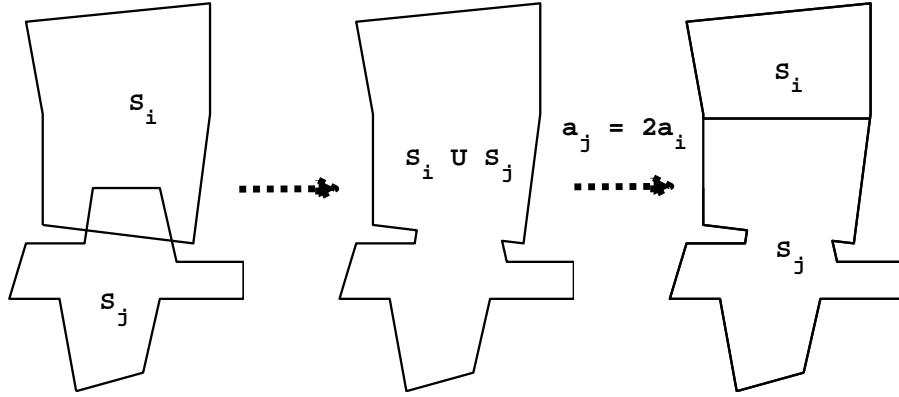


Fig. 9: Two aerial robots meet and share their actual covered areas. The whole area is divided between the robots according to their capabilities.

refresh time is increased. Minimizing that time, but ensuring synchronization, implies that each aerial robot patrols an area whose size is related to its own maximum capabilities (4). Computing an area division which accomplishes these conditions can be computationally expensive. Also, the obtained solution is not robust to changes in the robots' capabilities or area shape.

Algorithm 2 shows the *one-to-one* coordination technique which allows the robots self-adapt to cover an area according to their maximum capabilities and keep the synchronization in a distributed and decentralized manner. With the proposed technique, each aerial robot only needs information from its neighbors to converge to the area partition. When a two robots are close enough (distance less than communication range  $R$ ) to establish a communication, they exchange the area that they are covering and their own maximum capabilities, and they execute a *share & divide* function. Namely, each aerial robot joins the two areas and divide it according to the capabilities

$$A_i = a_i \frac{A(S_i \cup S_j)}{a_i + a_j}, \quad (7)$$

using a vertical or horizontal line depending on the link index as Fig. 9 shows.

## 6 VALIDATION TESTS

The proposed techniques have been validated by using MATLAB simulations. Although these simulations have been run using the quad-rotor model in [1], the approach can be directly applied to any other type of rotatory wing UAV (small changes in the coverage path generation algorithm are required for fixed wing UAVs). More than 100 simulation runs have been executed with a communication range of 5 m, different number of robots (4-16) and with

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**Algorithm 2** One-to-one distributed coordination algorithm executed onboard each aerial robot  $Q_i$ .

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$Q_i$  receives the whole area to cover and its grid cell position  
 $Q_i$  computes the initial area division  
 $Q_i$  initializes its own variables  
 $Q_i$  generates its own coverage paths and starts to move  
**for all**  $t$  **do**  
     $Q_i$  follows its own path  
    **if**  $Q_i$  arrives to a link position **then**  
        **if** It is a link position without neighbor **then**  
             $Q_i$  recomputes the link position  
             $Q_i$  generates its own coverage path  
        **else**  
            **if**  $Q_i$  does not meet its neighbor **then**  
                 $Q_i$  waits a time  $T_w$   
                 $Q_i$  joins a portion of the neighbor  $Q_j$  area to its own coverage area  
                 $Q_i$  recomputes the link positions  
                 $Q_i$  generates its own path  
                **if** Neighbor area size is zero **then**  
                     $Q_i$  labels this link positions as without neighbor  
                **end if**  
            **else**  
                 $Q_i$  receives information from neighbor  $Q_j$   
                 $Q_i$  executes a *share&divide* function  
                 $Q_i$  recomputes the common link position  
                 $Q_i$  generates its own coverage path  
            **end if**  
          **end if**  
        **end if**  
    **end for**

---

different area shapes. The maximum speed and field of view (FOV) for each UAV have been randomly chosen using a normal distribution: from 0.2 to 0.5 m/s for the speed and from  $\pi/8$  to  $\pi/6$  rad for the FOV. The next sections describe the features of the proposed algorithms.

### 6.1 Convergence under dynamic changes

This section summarizes the convergence metrics computed from the performed tests. It is assumed that a system has converged when the maximum difference between the optimum defined in (4) and the actual sub-area sizes for any robot is lower than 1 %. As the simulations consider different shapes and different UAV capabilities, a normalized convergence time  $\overline{T}_c$  is defined as the relation between the convergence time  $T_c$  and the average time that an UAV in the team would need to cover the whole area:

$$\overline{T}_c = \frac{T_c \sum_{j=1}^N a_j^{\max}}{NA}, \forall i = 1, \dots, N, \quad (8)$$

where  $N$  is the number of UAVs,  $A$  is the size of the area and  $a_j$  is the coverage speed for the  $j$ -th UAV as it is defined in (2).

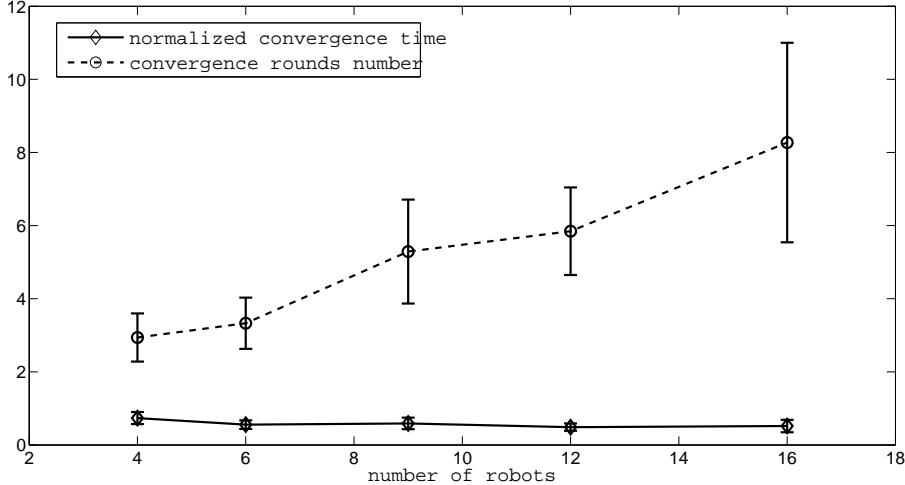


Fig. 10: Average normalized convergence times  $\bar{T}_c$  and the average number of rounds that each UAV needs to converge to the solution ( $\pm$  its standard deviation) with respect to the number of UAVs.

Figure 10 shows the average normalized convergence times and the average number of rounds that each UAV needs to converge to the solution ( $\pm$  its standard deviation) with respect to the number of UAVs. From the results it can be seen that as the number of robots increases, each robot needs a larger number of rounds to converge to the solution. However, as the round length is decreasing with the number of robots, the required time to converge remains almost constant.

In the following, a case of study is presented to validate the solution convergence in dynamic scenarios (area and UAV capabilities changes). A crowded square of  $1696.6\text{ m}^2$  in front of a church has to be monitored by a team of six aerial robots, see Fig. 11. The aerial robots have different capabilities, see Table 1.

Robot id.	1	2	3	4	5	6
Cell	1	2	3	4	5	6
Speed (m/s)	0.5	0.3	0.4	0.4	0.5	0.4
FOV (rad)	$\pi/8$	$\pi/6$	$\pi/8$	$\pi/7$	$\pi/6$	$\pi/7$

Table 1: Robot capabilities.

At time  $t = 600\text{ s}$ , the FOV of robot 5 is reduced to  $\pi/8\text{ rad}$  and, at time  $t = 1200\text{ s}$ , the place begins to clear and the area to cover is decreased to  $1406\text{ m}^2$ . Figure 12 shows the actual area size covered and the optimal one



Fig. 11: Snapshot taken from simulations in the dynamic case scenario. A video about this simulation is shown in <http://www.youtube.com/watch?v=t03kSWQ79J4>

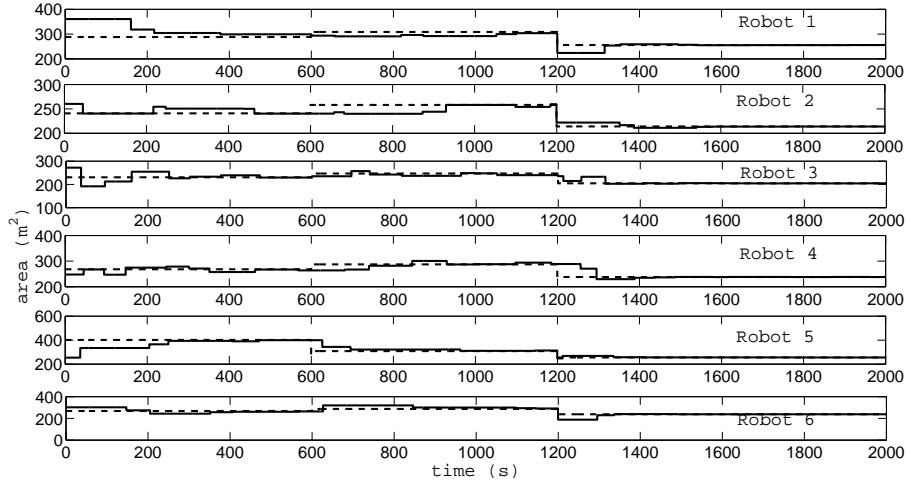


Fig. 12: Area size actually covered for each aerial robot (solid line) along the time and optimal area size that each robot should cover (dashed line).

according to expression (4) for each UAV along the time. It shows how fast the system is able to adapt to changes and converges to the optimal division in a distributed manner. Figure 13 shows the actual area division at 4 different times:  $t = 0\text{ s}$ ,  $t = 500\text{ s}$ ,  $t = 1000\text{ s}$  and  $t = 2000\text{ s}$ . Finally, Fig. 14 shows that two neighbor robots meet periodically achieving the intended synchronization.

## 6.2 Minimizing the information propagation time

In this section, metrics related to information detection and information propagation times are presented. It is assumed that the UAVs can detect any threat into the coverage range of their on-board sensors. For each setup, up

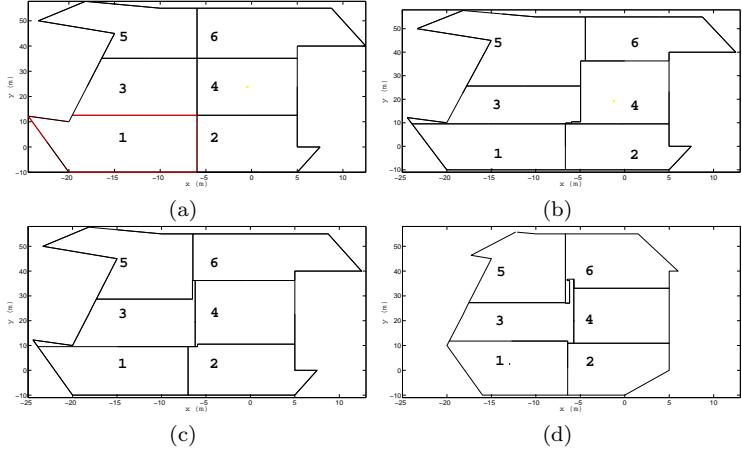


Fig. 13: This figure shows the area division between the six UAVs with heterogeneous capabilities at four different times: (a)  $t=0$  s, (b)  $t=500$  s, (c)  $t=1000$  s and (d)  $t=2000$  s.

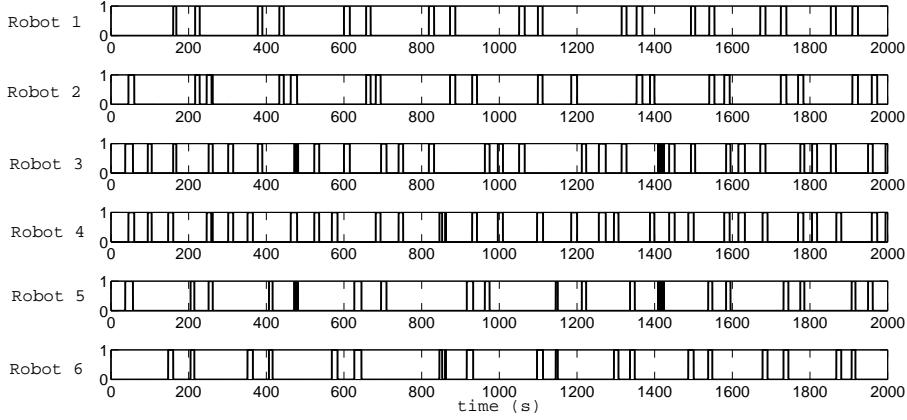


Fig. 14: This figure shows the time slots when each robot is contacting (within communication range) with another robot.

to 10 different threats are simulated at random locations. The detection time  $T_d$  is defined as the time since a threat appears until any UAV detects it. The propagation time  $T_p$  is the time since any UAV detects a threat until the rest of UAVs receive the information about it. As the areas and UAV capabilities are different for each test, normalized detection and propagation times have been defined as the relation between the measured times and the average time that an UAV in the team would take to cover the whole area:

$$\overline{T_d} = \frac{T_d \sum_{j=1}^N a_j^{\max}}{NA}, \forall i = 1, \dots, N \quad (9)$$

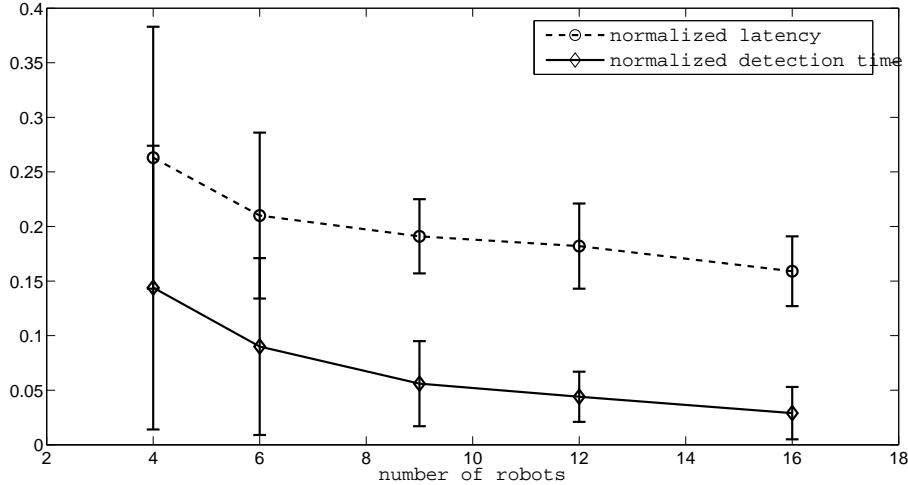


Fig. 15: Average normalized detection  $\overline{T}_d$  and propagation  $\overline{T}_p$  times with respect to the number of UAVs according to the simulation results.

$$\overline{T}_p = \frac{T_p \sum_{j=1}^N a_j^{\max}}{NA}, \forall i = 1, \dots, N \quad (10)$$

where  $N$  is the number of UAVs,  $A$  is the area size and  $a_j$  is the coverage speed for the  $j$ -th UAV in (2).

Figure 15 shows the average normalized detection and propagation times ( $\pm$  its standard deviation) with respect to the number of UAVs. In this figure it can be seen that as the number of robot increases, the detection and sharing information performance is improved.

### 6.3 Comparison with other patrolling approaches

The selected scenario for the comparison is a city where a team of 16 UAVs has monitor downtown, see Fig. 16, in order to detect dangerous situations or suspicious people and report to the police station (located at position [50, 35] m). Figure 17 shows as the proposed area division system converges to the partition which theoretically minimizes the refresh time and the latency. Figure 18 shows two different actual area divisions at time  $t = 0$  s and at time  $t = 2500$  s.

It is assumed that the communication range is limited to 10 m. The whole area to monitor has a size of  $65517 \text{ m}^2$  and all the UAVs have the same coverage capabilities (such as it can be properly compared with the cyclic strategy): a maximum speed of 1 m/s and a FOV of  $\theta = \pi/4$  rad. Their flight altitude is 6 m and each UAV would take a time of  $T_v = 5459.7$  s to patrol the whole area. In the tests up to 50 different threats were simulated at random positions.

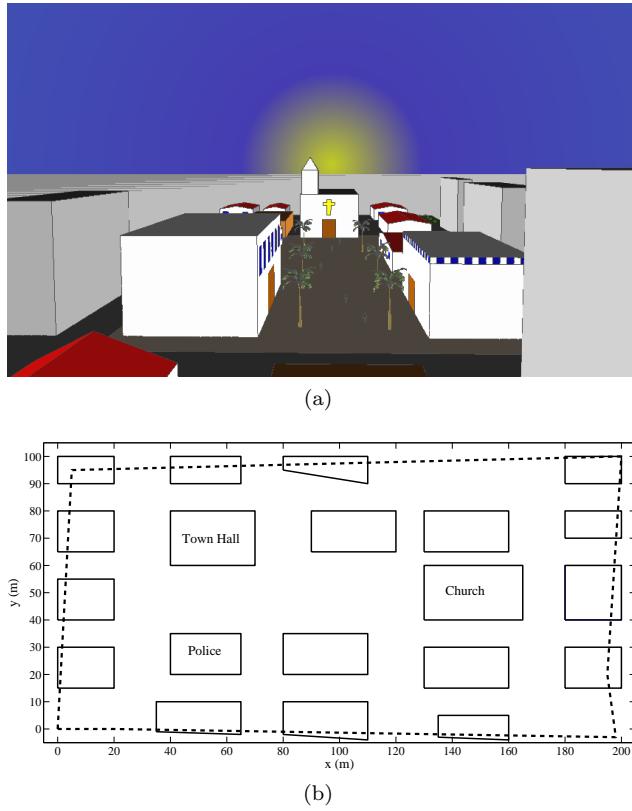


Fig. 16: Snapshot (a) and area map (b) used in the MATLAB simulation with 16 aerial robots covering downtown in a urban scenario.

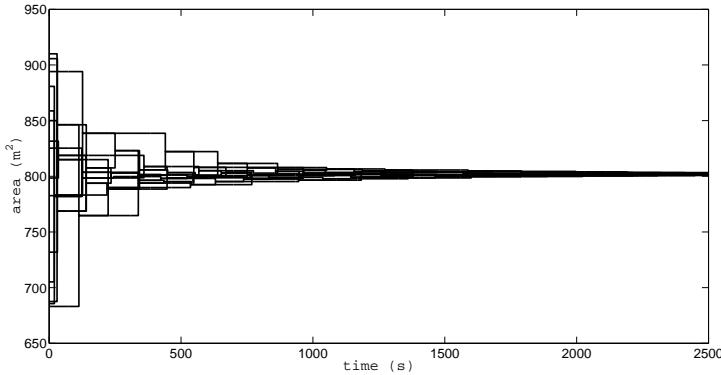


Fig. 17: Actual area size patrolled for each aerial robot along the time. The system converge to an equally divided area because the robots are homogeneous.

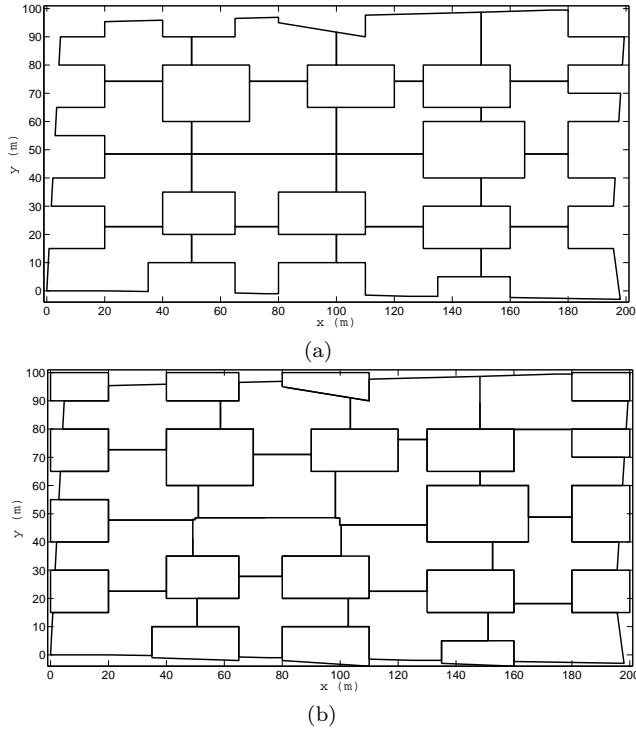


Fig. 18: The actual area divisions with the 16 aerial robots at times: (a)  $t = 0$  s and (b)  $t = 2500$  s. The police station antenna receiver is located in the coordinates  $(50, 35)$  m.

The methods presented in this paper are compared with the cyclic and path partition strategies:

- The cyclic strategy assumes that all the UAVs move in the same direction following the same closed path and equally spaced. UAVs do not exchange information between them. Therefore, if any UAV detects any threat, it has to arrive close enough to the police station to report it.
- In the path partitioning strategy proposed in [1] the UAVs divide a single coverage path into segments. Thus each UAV patrols a different segment and UAVs meet in their common endpoint segments to share information.

Figure 19 shows the average detection time and time to inform the police station ( $\pm$  its standard deviation) for the four different strategies. Results show that, although the three strategies obtain similar detection times, the proposed method in this paper obtains the best detection and report time performance (at least three times lower).

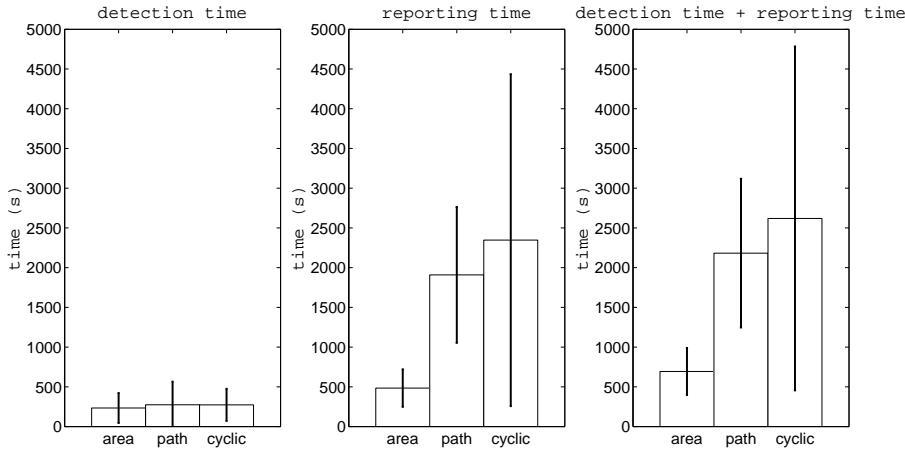


Fig. 19: Detection times, times to inform police station and sum of both during simulations, considering: area partitioning, path partitioning and cyclic strategies. It shows the average times ( $\pm$  its standard deviation).

## 7 CONCLUSIONS

This paper proposed an area partitioning strategy to solve the problem for irregular areas and heterogeneous UAVs. The whole area is divided into non-overlapping sub-areas, each one monitored by an aerial robot using an efficient path, i.e. all the positions in the area are observed while the path is traveled, minimizing the total path length. Each robot covers an area with a size related to its motion and sensing capabilities, minimizing the time to cover the whole area.

A *one-to-one* coordination technique allows to redistribute the area between the aerial robots in a decentralized and distributed manner in order to obtain a more efficient area division. The proposed coverage path planning algorithm, where the distance between each pair of link positions is the same, allows to keep the synchronization between the robots.

Simulation results show a scalable solution which converges to an efficient area division (according to the capabilities of the aerial robots) and is able to adapt to changes in the initial conditions (area shape, robots capabilities), even with short communication ranges. Furthermore, results show as the detection time and the latency decrease as the number of aerial robots increases. Finally, comparisons with other strategies (path-partition and cyclic strategies) show that the proposed approach offers a better performance to detect threats and share information about them.

## ACKNOWLEDGMENTS

The synchronization problem studied here was introduced and partially solved during the VI Spanish Workshop on Geometric Optimization, June 2012, El Rocío, Huelva, Spain. The authors would like to thank other participants for helpful comments.

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## RESPONSES TO REVIEWERS (references and citations in this document are related to presented paper)

This paper extends the previous work presented in [2] providing a deeper mathematical analysis of the main algorithm and additional results, addressing the problem in urban environments. All reviewer comments from that previous work have been incorporated in this enhanced version.

Reviewer comments are highlighted using italic style. Responses to reviewers are highlighted using bold style.

### **REVIEW 1049:**

*The paper is well written and presents a good theoretical contribution.*

**Thanks a lot.**

*The authors should emphasize further steps with respect to the work "Cooperative large area surveillance with a team of aerial mobile robots for long endurance missions" [3] try to summarize differences with this one-to-one coordination technique here proposed.*

**This paper addresses the problem assuming heterogeneous robots and irregular areas. Also, the synchronization and information propagation between robots is addressed explicitly, providing the necessary conditions to guarantee it.**

*Please check reference order and formal punctuation.*

**Checked and corrected.**

*Please clarify why a decentralized approach is necessary to redistribute the area between the UAVs in a specific application like pollution monitoring.*

**This new paper addressed the problem in different scenarios (urban scenario). However, this issue can be responded in the same manner.**

**In the sixth paragraph of the Introduction Section, this issue is clarified "A decentralized approach offers robustness and dynamism, in a way that each UAV can quickly self-adapt its sub-area. Therefore, the system is able to perform the surveillance mission in the more efficient manner, even if the communication link with the control station is broken".**

### **REVIEW 1359:**

*1.a Please provide explanation about the real world problem this work is trying to address. Is there any real case of pollution source search (or any other monitoring task) performed by swarms of UAVs ?*

**This new paper addresses the problem in a different scenario (urban scenario) . In the first paragraph at Introduction Section, the applications and scenarios considered in the EC\_SAFEMOBIL research project are highlighted and related to the presented work**

*1.b Even if the problem is theoretical there are examples in Accademia about live experiments with swarms of robots (for example ground robots). Are they being considered?*

**A large set of reference has been considered and referenced in this paper. Some of them, as [16], [17] or [18], presents live experiments with multi-UAV systems. Any other related reference would be welcome.**

*2. Which UAV segment is being considered? Even if the algorithm is general, simulations are performed with dimensional values which do not suggest existing UAVs (such as maximum speed 1 m/s, communication range 10m, etc)*

**The simulations have been implemented using a quad-rotor model. However, the system is useful for any kind of rotatory wing UAV. A sentence in the first paragraph of the Validation section tries to clarify this issue: “Although these simulations have been run using the quad-rotor model in [1], the approach can be directly applied to any other type of rotatory wing UAV (small changes in the coverage path generation algorithm are required for fixed wing UAVs).”**

**The communications range has been selected to validate the theoretical advantages of the system under communications constraints.**

*3. Some constraints are neglected, some of which can influence the validity of the conclusions. For example:*

- endurance of each UAV*
- distance to Control Station*

*- sensor used to detect pollution: this can alter very much the search strategy, depending on the type of sensor (EO/IR/Multispectral/SAR can be oriented, thus gaining coverage), while "sniffers" or "artificial noses" need to cross the polluted cloud.*

**The endurance of each UAV is relevant. However, the system is decentralized and distributed and can self-adapt to changes. Therefore, if an UAV has to leave the mission, the rest of the rest of the team could cover its sub-area. It is highlighted in the third paragraph of the Introduction section.**

**The distance to Control Station is not relevant as long as it is into the monitored area. In figure 16, the caption has been modified to clarify that the control station receiver is into the monitored area.**

**In this paper, the only sensor relevant feature is the coverage range, such as it is shown in the expression (1).**

**We would like to thank reviewer comments and suggestions because they have helped us to improve our work.**